



METODOLOGIJA NAUČNOG ISTRAŽIVANJA

# NORMALNA RASPODELA

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FAKULTET ZA SPECIJALNU EDUKACIJU I REHABILITACIJU  
MASTER AKADEMSKE STUDIJE



# BACANJE NOVČIĆA



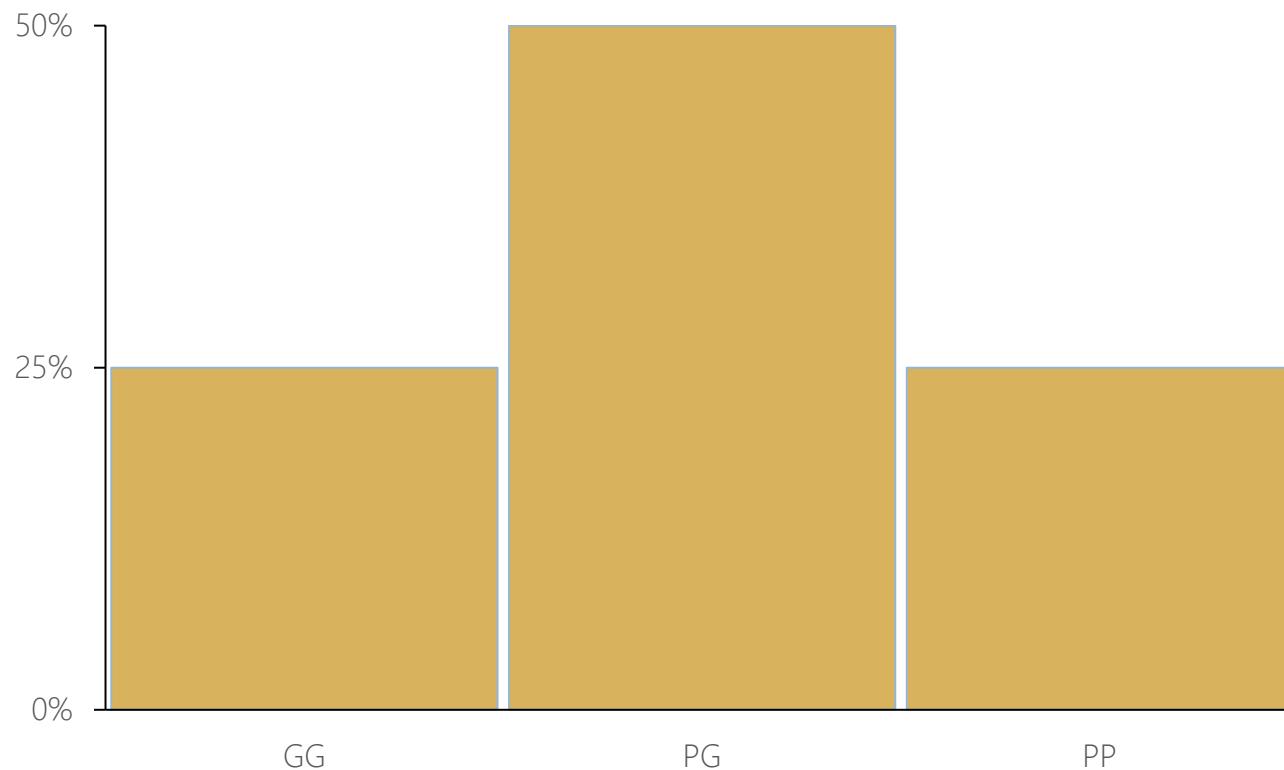
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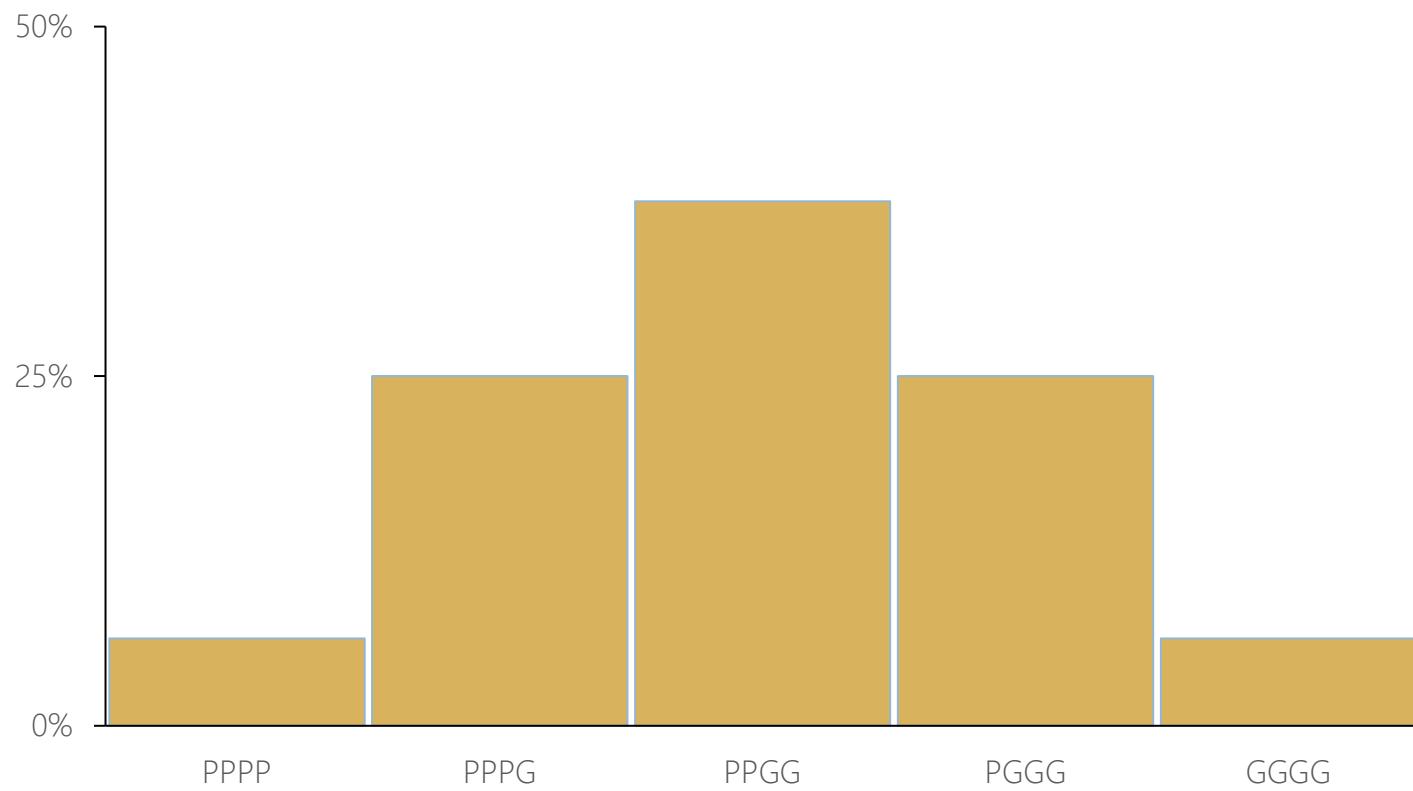


# DVA BACANJA NOVČIĆA



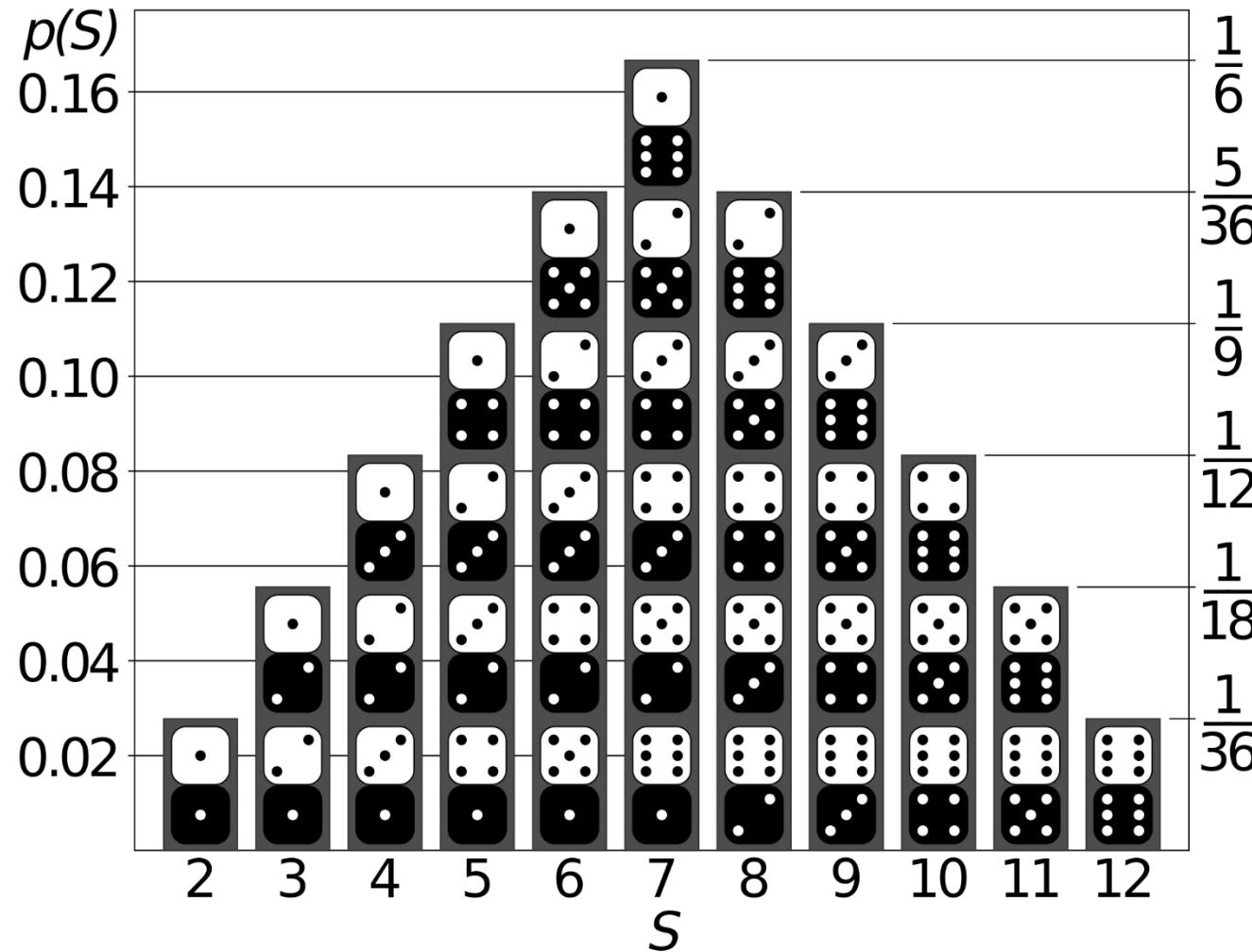


# ČETIRI BACANJA NOVČIĆA





# ZBIR DVE KOCKICE



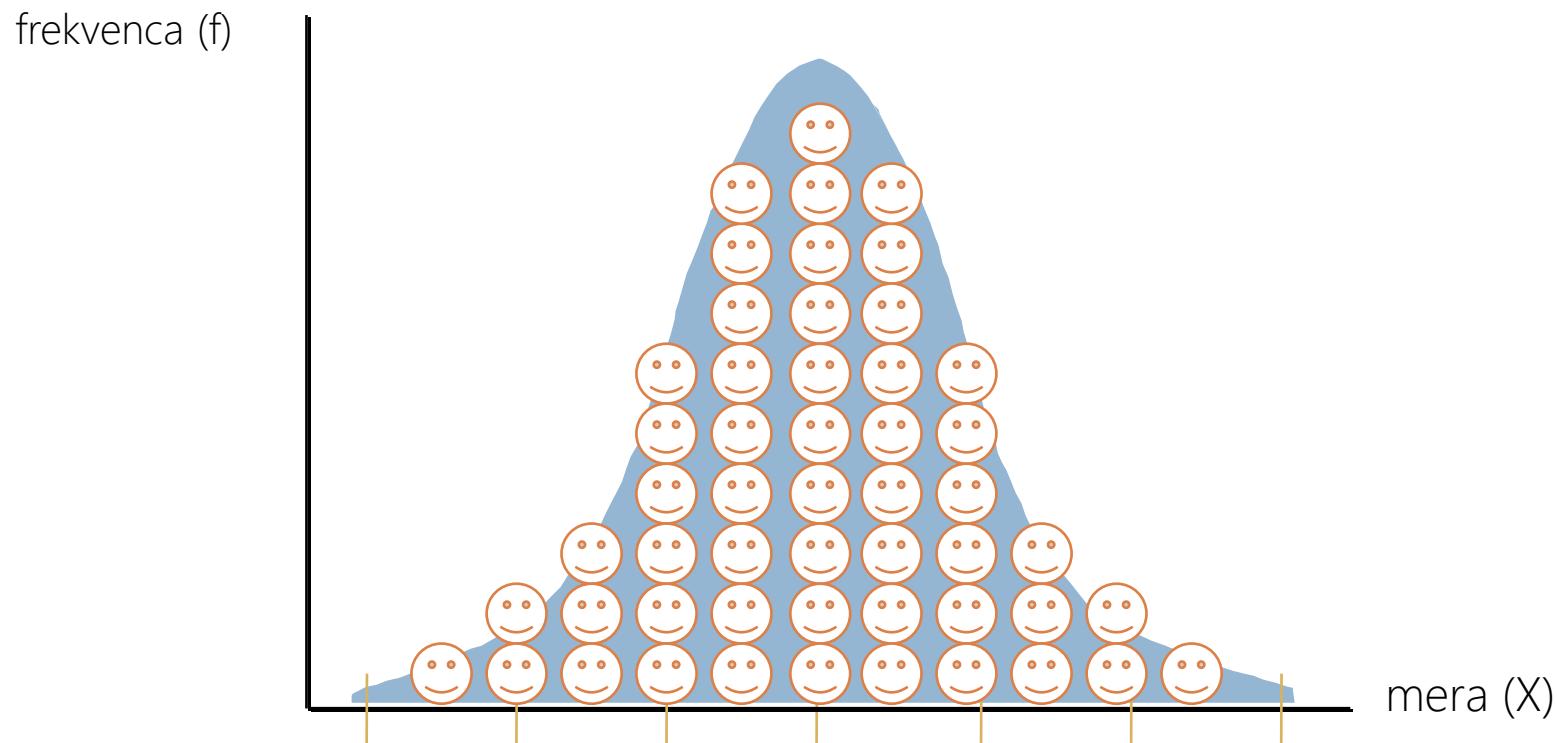


# NORMALNA RASPODELA UČESTALOSTI MERA





# NORMALNA RASPODELA UČESTALOSTI MERA

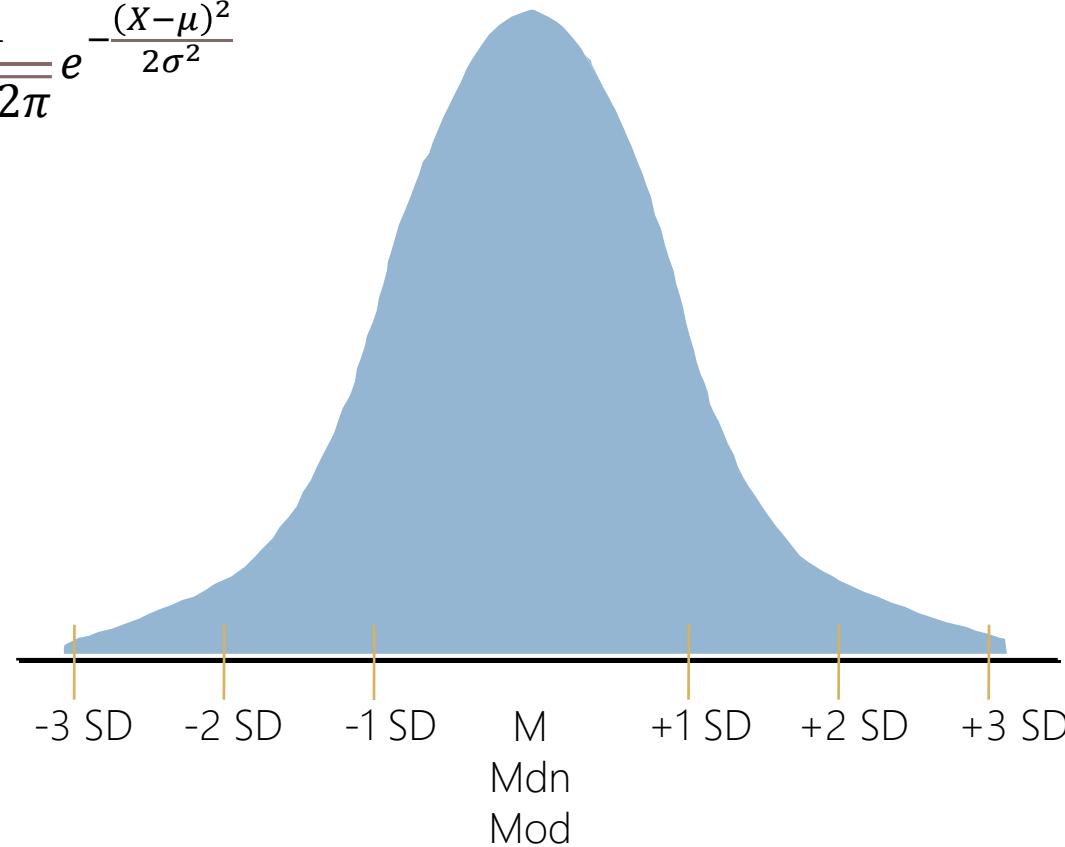


visina	145	155	165	175	185	195	205
otkucají srca	65	70	75	80	85	90	95
skor na testu	2	5	8	11	14	17	20



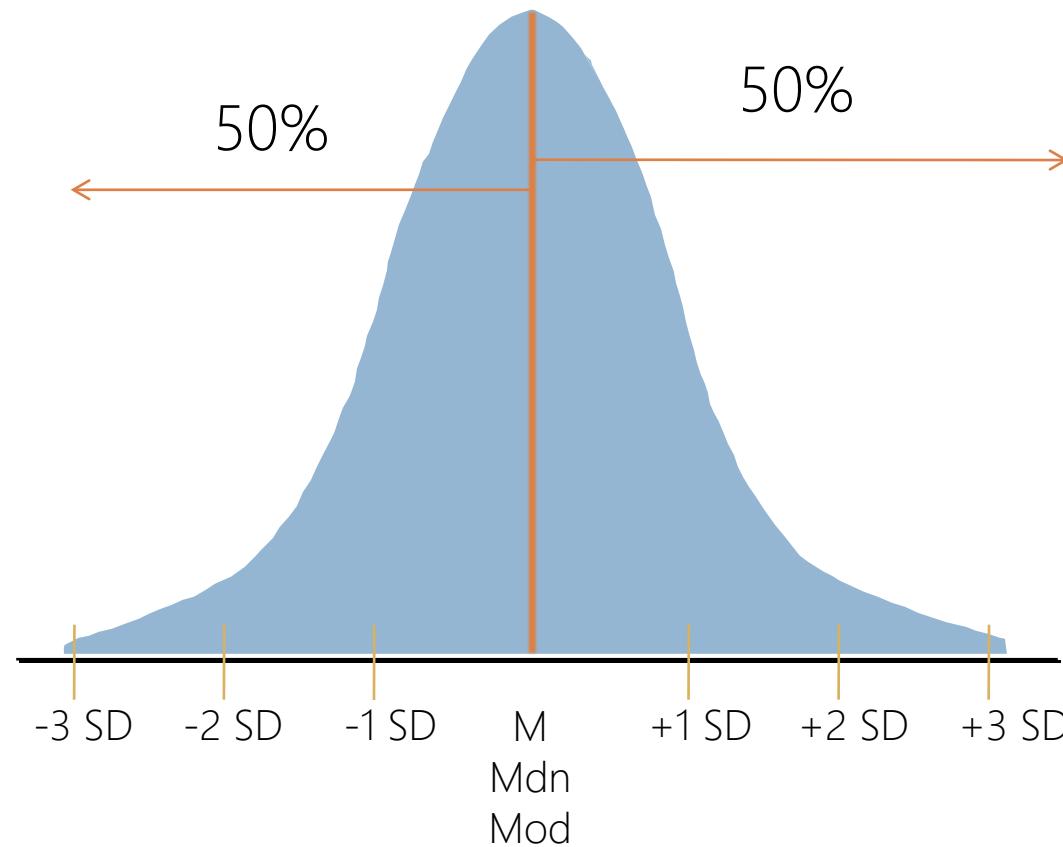
# NORMALNA RASPODELA

$$f_{(X)} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(X-\mu)^2}{2\sigma^2}}$$



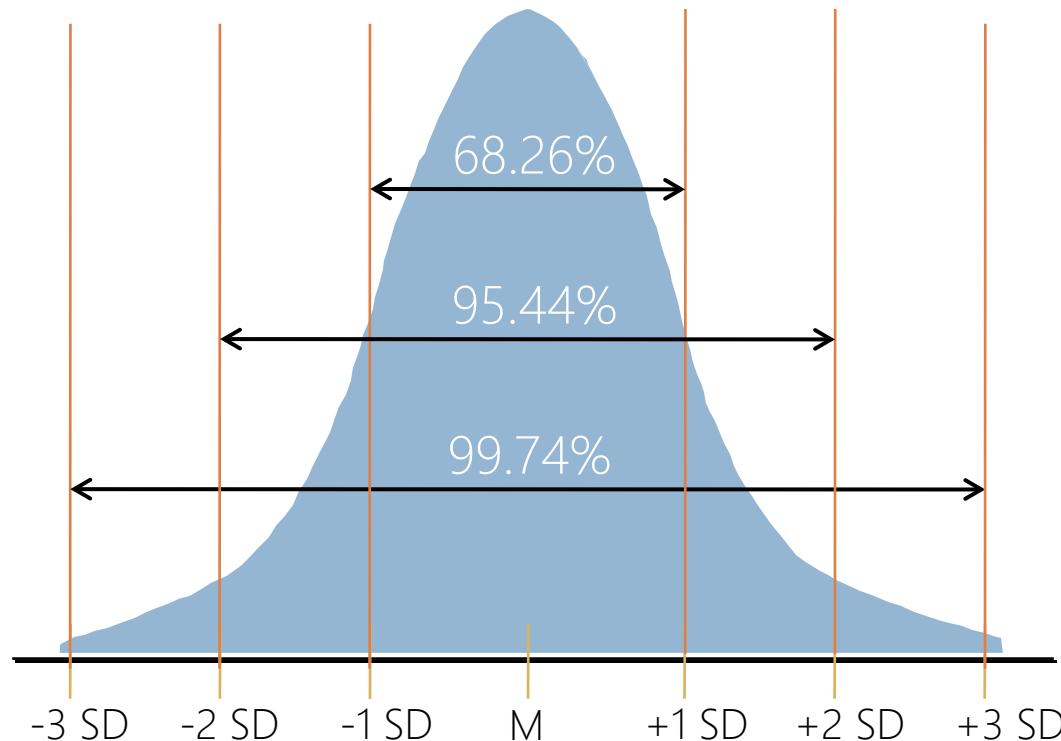


# NORMALNA RASPODELA



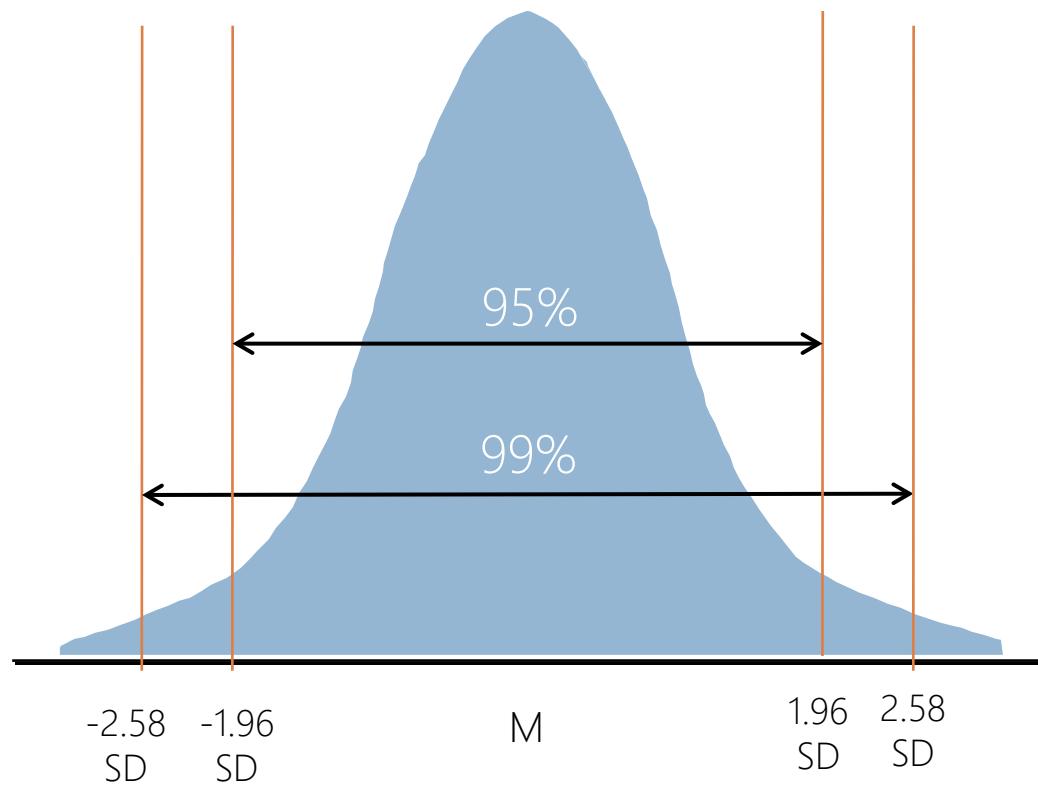


# POVRŠINA POD NORMALNOM KRIVOM





# POVRŠINA POD NORMALNOM KRIVOM





# PRIKAZ RASPODELE (HISTOGRAM)

- 1 Analyze → Descriptive Statistics → Frequencies → Chart → Histograms
- 2 Označiti opciju Show normal curve on histogram

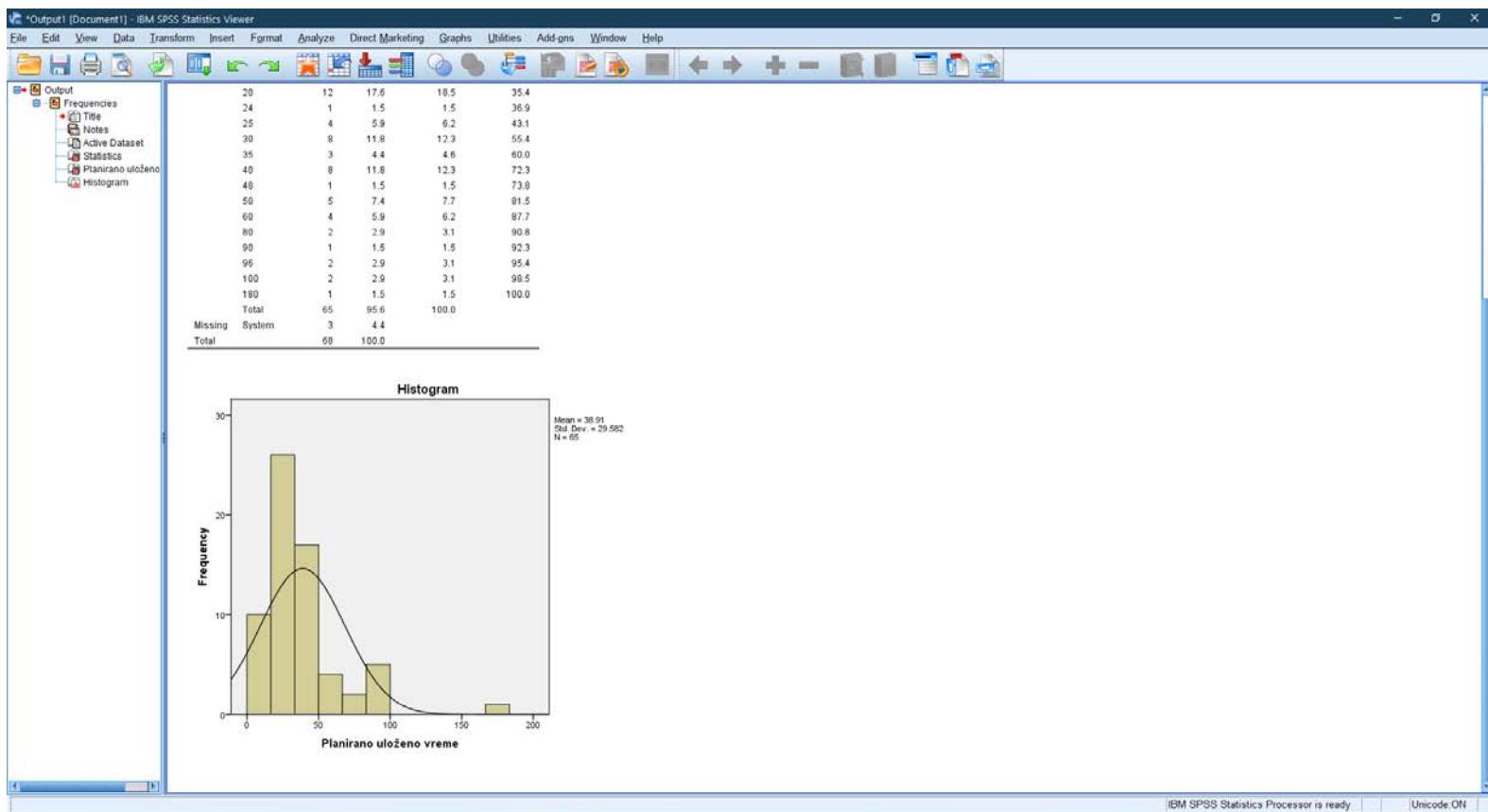
The screenshot shows the IBM SPSS Statistics Data Editor interface. The menu bar at the top includes File, Edit, View, Data, Transform, Analyze, Direct Marketing, Graphs, Utilities, Add-ons, Window, and Help. The 'Analyze' menu is highlighted, and a sub-menu is open under 'Descriptive Statistics' with 'Frequencies' selected. A red number '1' is placed over the 'Frequencies' option. Below the menu, the 'Variable View' is active, showing a list of variables: pol, smer, i1, i2, i3, i4, i5, i6, and ocena. The 'Data View' tab is also visible at the bottom left.

A 'Frequencies' dialog box is open in the center. In the 'Variable(s)' section, 'vreme' is selected. A 'Frequencies: Charts' sub-dialog is open over the main dialog. In this sub-dialog, the 'Chart Type' section has 'Histograms' selected with a radio button, and the 'Show normal curve on histogram' checkbox is checked, indicated by a red number '2'. Other chart type options like 'Bar charts', 'Pie charts', and 'None' are available but not selected. The 'OK' button is visible at the bottom right of the sub-dialog.

At the bottom of the screen, status bars show 'IBM SPSS Statistics Processor is ready' and 'Unicode:ON'.



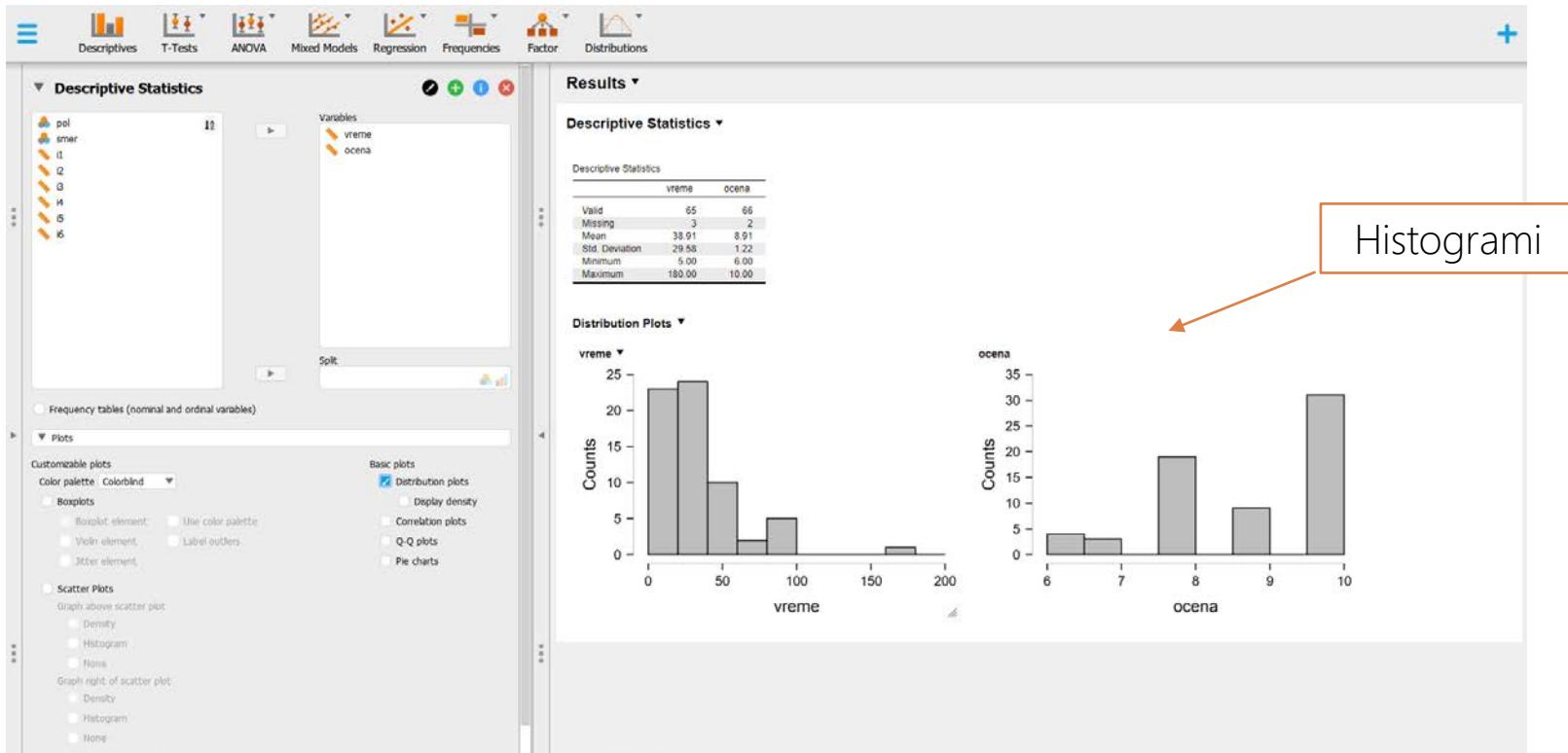
# PRIKAZ RASPODELE (HISTOGRAM)



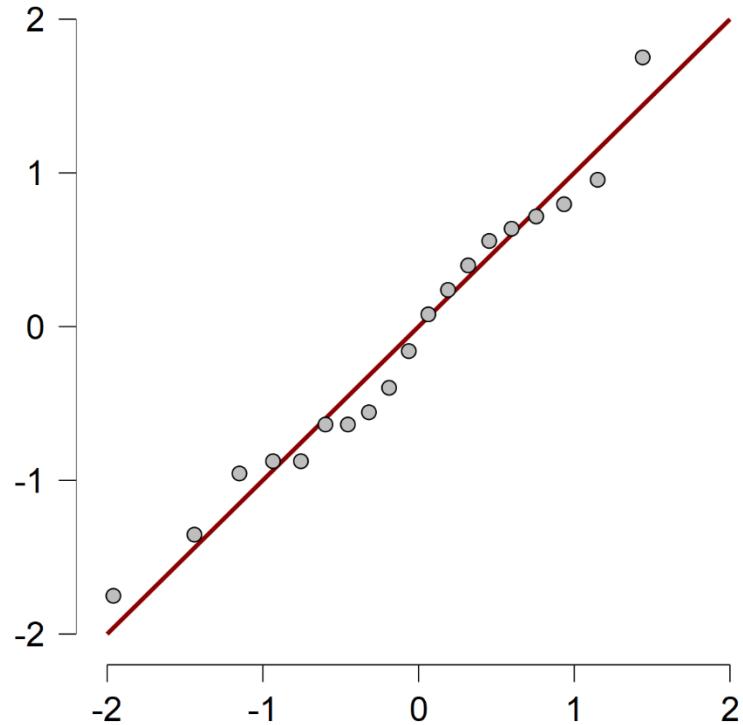


# PRIKAZ RASPODELE (HISTOGRAM)

U podmeniju Descriptives - Plots odabratи Distribution plots



Napomena: označavanjem opcije **Display density** može se dobiti grafički prikaz krivulje raspodele



# OMNIBUS TESTOVI NORMALNOSTI RASPODELE



# Q-Q PLOT

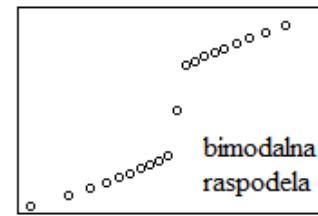
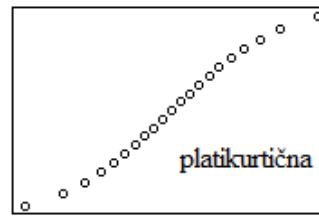
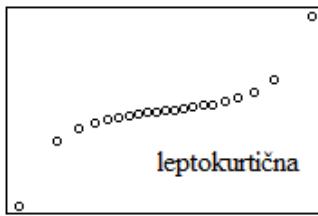
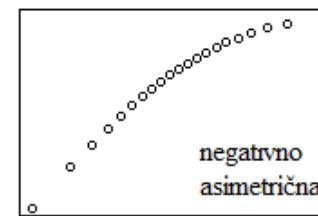
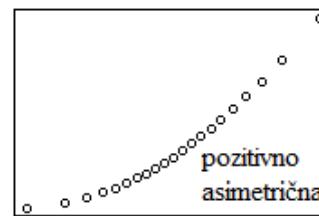
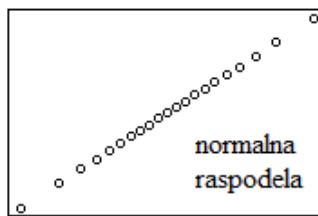
quantile-quantile plot (eng.)

Grafički metod za poređenje dve distribucije (teorijske i empirijske)

Teorijska distribucija je po pravilu normalna (mada se mogu plotovati i druge)

Ukoliko se dve distribucije poklapaju, tačke će ležati na pravoj liniji

Ukoliko postoji odstupanje, tačke će odstupati od prave linije (kao u primeru sa slike)





# OMNIBUS TESTOVI NORMALNOSTI RASPODELE

## Kada se koriste omnibus testovi normalnosti raspodele?

Kada nas zanima da li empirijska raspodela statistički značajno odstupa od normalne.

## Koju nultu hipotezu ispituju omnibus testovi normalnosti raspodele?

$H_0$ : Empirijska raspodela nije različita u odnosu na normalnu raspodelu.

## Koje odluke možemo da donešemo na osnovu rezultata testa?

Ako je  $p > .05$  onda ne odbacujemo nultu hipotezu

tj. zaključujemo da empirijska raspodela ne odstupa značajno od normalne

Ako je  $p < .05$  onda odbacujemo nultu hipotezu

tj. zaključujemo da empirijska raspodela odstupa značajno od normalne

## Koji sve omnibus testovi normalnosti postoje?

Kolmogorov-Smirnov test

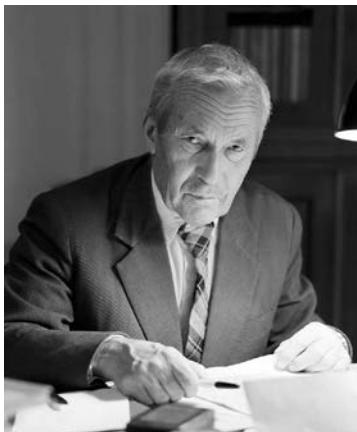
Šapiro-Vilk test

Anderson-Darling test

Lillefors



# KOLMOGOROV-SMIRNOV TEST



Андрей Колмогоров  
(1903 - 1987)

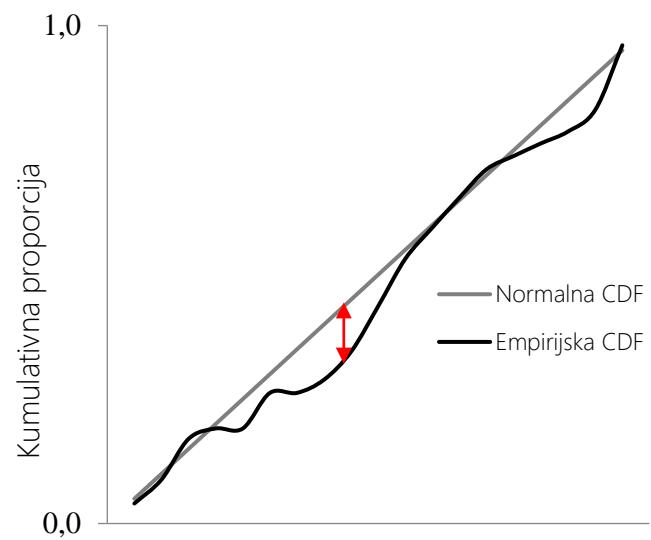


Николай Смирнов  
(1900 - 1966)

$$D = \max |F_t(X) - F_e(X)|$$

## Šta radi Kolmogorov-Smirnov test?

Ispituje da li je maksimalno odstupanje kumulativne empirijske frekvence u odnosu na modelom normalne raspodele očekivanu kumulativnu frekvencu dovoljno veliko da bi se moglo smatrati statistički značajnim.

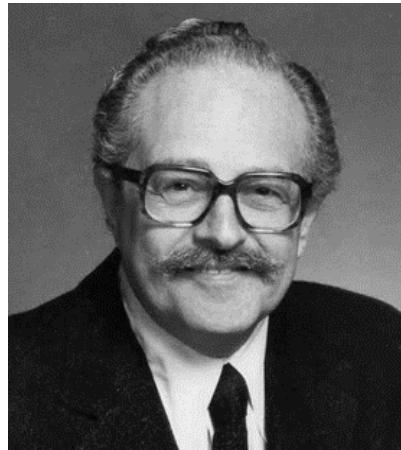




# ŠAPIRO-VILK TEST



Samuel Shapiro  
(1930 - )



Martin Wilk  
(1922 - 2013)

$$W = \frac{\left(\sum_{i=1}^n a_i X_{(i)}\right)^2}{\sum_{i=1}^n (X_i - M)^2}$$

## Šta radi Šapiro-Vilk test?

Iskazuje stepen sličnosti (preklapanja) empirijske i teorijske (normalne) distribucije putem jednog broja.



# TESTOVI NORMALNOSTI RASPODELE

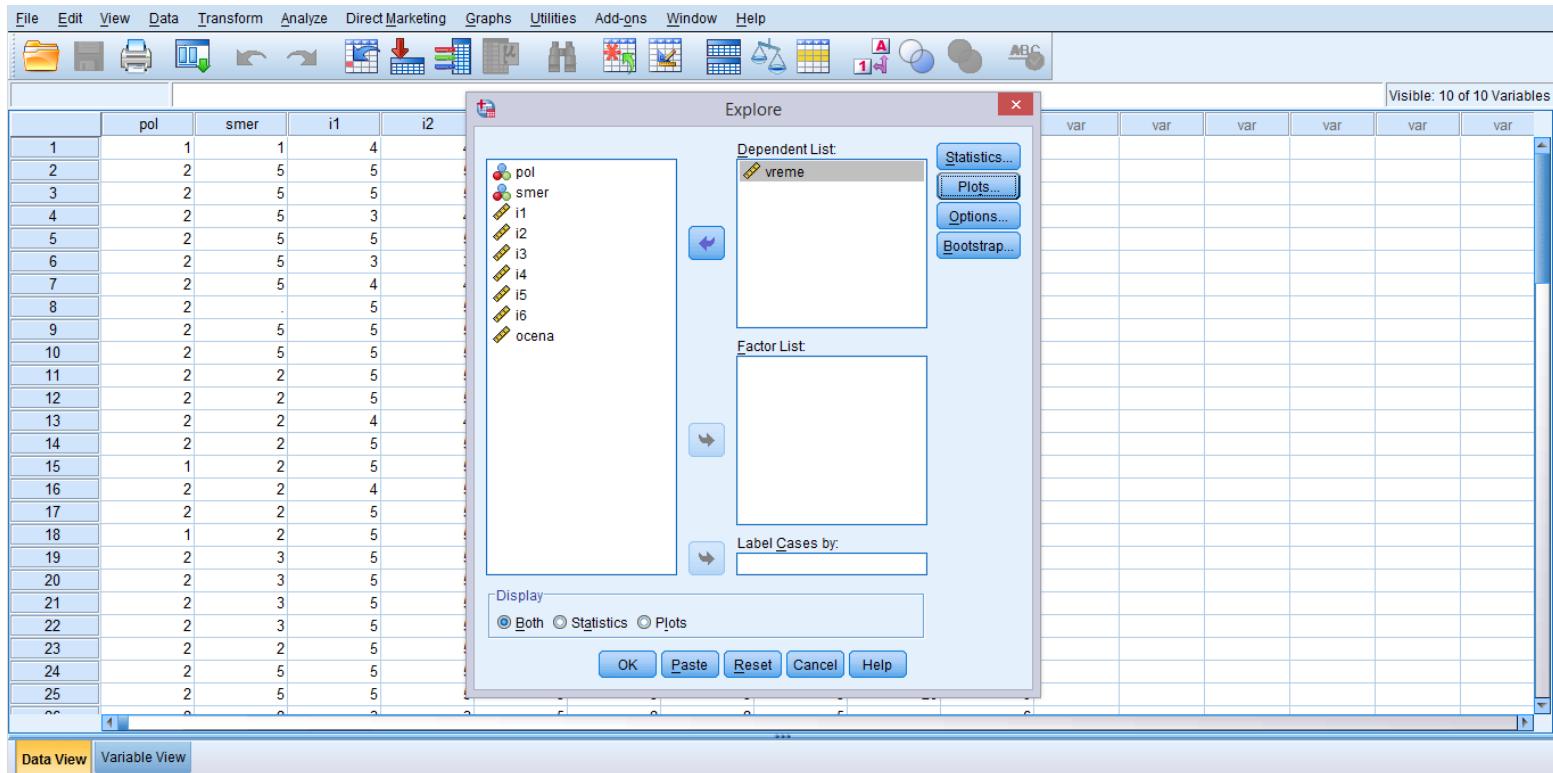
## Analyze → Descriptive Statistics → Explore

The screenshot shows the SPSS software interface. The top menu bar includes File, Edit, View, Data, Transform, Analyze, Direct Marketing, Graphs, Utilities, Add-ons, Window, and Help. The Analyze menu is open, revealing various statistical options. Under the Descriptive Statistics category, 'Explore...' is highlighted. A data grid below shows variables i5, i6, vreme, ocena, and var across 25 observations. The status bar at the bottom indicates 'Visible: 10 of 10 Variables'.

	pol	smer	i5	i6	vreme	ocena	var	var	var	var	var	var
1	1		1	5	-	8						
2	2		3	3	20	10						
3	2		3	5	20	10						
4	2		4	4	20	8						
5	2		4	5	60	9						
6	2		3	5	20	6						
7	2		4	3	5	24	8					
8	2		5	4	5	80	10					
9	2		5	1	3	180	10					
10	2		5	4	5	40	9					
11	2		5	5	1	50	10					
12	2		5	2	4	80	10					
13	2		5	4	4	2	90	10				
14	2		5	5	5	15	10					
15	1		5	3	3	20	10					
16	2		5	3	3	30	8					
17	2		5	5	5	15	10					
18	1		5	4	2	40	10					
19	2		5	5	3	100	10					
20	2		5	3	1	5	60	10				
21	2		5	3	3	100	10					
22	2		5	3	3	4	9					
23	2		5	1	1	5	8					
24	2		3	5	4	50	10					
25	2		5	3	3	20	9					

# TESTOVI NORMALNOSTI RASPODELE

Prebaciti željenu/e varijable u [Dependent List](#) prozor. OK



Napomena: Moguće je istovremeno testirati normalnost u više grupa.  
U tom slučaju treba prebaciti i kategoričku varijablu u [Factor List](#) prozor.

# TESTOVI NORMALNOSTI RASPODELE

U podmeniju **Plots** isključiti **Steam-and-leaf**, a uključiti **Normality plots with tests**

The screenshot shows the SPSS interface with the 'Data View' selected. A dialog box titled 'Explore' is open, with 'vreme' selected in the 'Dependent List'. A sub-dialog titled 'Explore: Plots' is also open, showing the following settings:

- Boxplots:** Factor levels together (selected)
- Descriptive:** Stem-and-leaf (unchecked), Histogram (checked)
- Normality plots with tests:** Checked
- Spread vs Level with Levene Test:** None (selected)
- Power estimation:** Available but not selected
- Transformed Power:** Natural log (selected)

At the bottom of the 'Explore: Plots' dialog are 'Continue', 'Cancel', and 'Help' buttons.

# TESTOVI NORMALNOSTI RASPODELE

**Tests of Normality**

	Kolmogorov-Smirnov <sup>a</sup>			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
vreme Planirano uloženo vreme	.208	65	.000	.784	65	.000

a. Lilliefors Significance Correction

Rezultati K-S testa  
(uz Lilliefords korekciju)

Rezultati  
Shapiro-Wilk testa



# TESTOVI NORMALNOSTI RASPODELE

Najprej je potrebno uključiti meni **Distributions** klikom na znak + u gornjem desnom uglu

The screenshot shows the SPSS Statistics software interface. At the top, there is a toolbar with various statistical analysis icons: Descriptives, T-Tests, ANOVA, Mixed Models, Regression, Frequencies, Factor, and Distributions. The 'Distributions' icon is highlighted with a blue border. Below the toolbar is a menu bar with several options like Audit, Bain, Distributions (which is checked), Equivalence T-Tests (Beta), JAGS, Machine Learning, Meta-Analysis, Network, Reliability, SEM, Summary Statistics, Visual Modeling, Learn Bayes, and R (Beta). On the left side of the main window, there is a data view showing a table with 30 rows and 10 columns. The columns are labeled: pol, smer, I1, I2, B, H, S, Vreme, and ocena. The data consists of numerical values ranging from 2 to 5. On the right side, there is a large gray area representing the output viewer where results would be displayed.

	pol	smer	I1	I2	B	H	S	Vreme	ocena
1	1	težo	4	4	5	1	1	5	8
2	2	prevencija	5	5	5		3	3	10
3	2	prevencija	3	5	4	3	3	5	20
4	2	prevencija	3	4	4	3	4	4	20
5	2	prevencija	5	5	5	4	4	5	60
6	2	prevencija	3	3	5	3	3	5	20
7	2	prevencija	4	4	4	3	3	5	24
8	2		5	5	5	4	4	5	80
9	2	prevencija	5	5	5	1	1	3	100
10	2	prevencija	5	5	5	4	4	5	40
11	2	somato	5	5	5	5	1	5	50
12	2	somato	5	5	5	2	4	5	80
13	2	somato	4	4	5	4	4	2	90
14	2	somato	5	5	5	5	5	5	15
15	1	somato	5	5	5	3	3	5	20
16	2	somato	4	5	5	3	3	5	30
17	2	somato	5	5	5	5	5	5	15
18	1	somato	5	5	5	4	2	5	40
19	2	senzo-moto	5	5	5	5	3	5	100
20	2	senzo-moto	5	5	5	3	1	5	60
21	2	senzo-moto	5	5	5	3	3	5	100
22	2	senzo-moto	5	5	5	3	3	4	30
23	2	somato	5	5	5	1	1	5	60
24	2	prevencija	5	5	3	5	4	4	50
25	2	prevencija	5	5	5	3	3	5	20
26	2	somato	3	3	5	2	2	5	6
27	2	senzo-moto	3	2	5	1	1	4	18
28	2	prevencija	5	5	5	3	3	3	50
29	1	somato	3	3	4	2	2	5	20
30	2	prevencija	4	5	5	4	4	5	50



# TESTOVI NORMALNOSTI RASPODELE

## Distributions – Continuous – Normal

Screenshot of a statistical software interface showing a data table and a distribution selection menu.

The data table has columns labeled: pol, smer, i1, i2, i3, i4, i5, i6, cena, and two empty columns at the end.

	pol	smer	i1	i2	i3	i4	i5	i6	cena		
1	1	tiflo	4	4	5	1	1	5			
2	2	prevencja	5	5	5			3	3		
3	2	prevencja	5	5	4	3	3	5			
4	2	prevencja	3	4	4	3	4	4			
5	2	prevencja	5	5	5	4	4	5			
6	2	prevencja	3	3	5	3	3	5			
7	2	prevencja	4	4	4	3	3	5			
8	2		5	5	5	4	4	5			
9	2	prevencja	5	5	5	1	1	3			
10	2	prevencja	5	5	5	4	4	5			
11	2	somato	5	5	5	5	1	5			
12	2	somato	5	5	5	2	4	5			
13	2	somato	4	4	5	4	4	2	90	10	
14	2	somato	5	5	5	5	5	5	15	10	
15	1	somato	5	5	5	3	3	5	20	10	
16	2	somato	4	5	5	3	3	5	30	8	
17	2	somato	5	5	5	5	5	5	15	10	

A dropdown menu titled "Continuous" is open, listing various probability distributions:

- Normal
- Student's t
- F-distribution
- Chi-squared
- Beta
- Gamma
- Inverse gamma
- Exponential
- Log-normal
- Logistic

A second dropdown menu titled "Discrete" is also visible, listing:

- Bernoulli
- Binomial
- Negative binomial
- Poisson



# TESTOVI NORMALNOSTI RASPODELE

U podmeniju [Generate and Display Data](#) prebaciti željenu varijablu u okvir Get variable from data set

The screenshot shows the configuration of a 'Normal' node in the SPSS Modeler interface. The node is expanded to show its options.

**Parameters:** Mean:  $\mu = 0$ , Variance:  $\sigma^2 = 1$

**Display:** Probability density function (selected), Cumulative distribution function, Quantile function.

**Options:** Range of  $x$  from -3 to 3, Highlight Density.

**Generate and Display Data:** Generate new variable from Normal ( $\mu = 0, \sigma^2 = 1$ ). Variable name: e.g., random Normal, Number of samples: 68, Draw samples button.

**Get variable from data set:** A list of variables: pol, smer, i1, i2, i3, i4, i5, i6, vreme. The variable 'ocena' is selected.

**Statistics:** Descriptives (selected), First 2 observed moments.

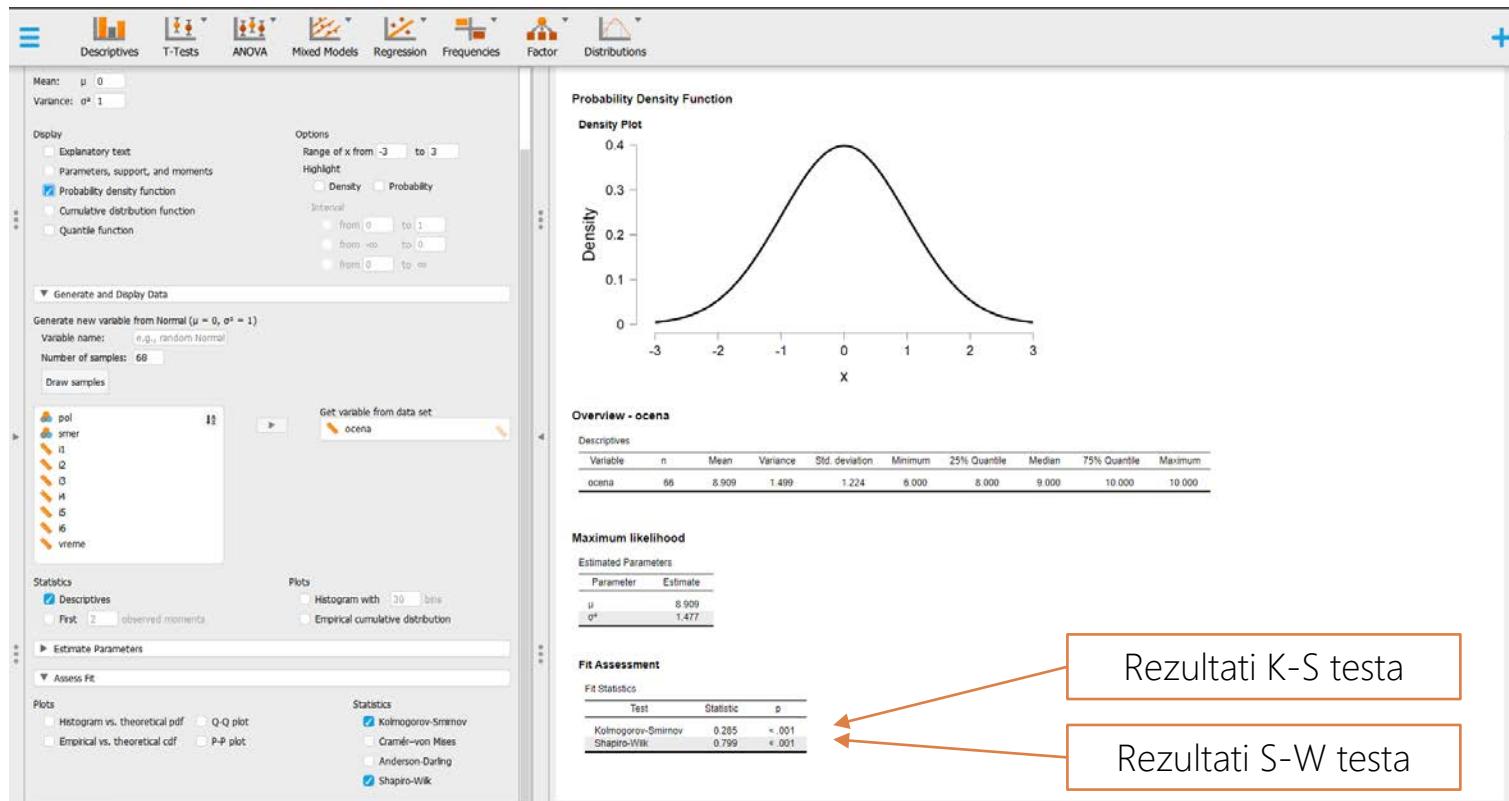
**Plots:** Histogram with 30 bins, Empirical cumulative distribution.

**Estimate Parameters** and **Assess Fit** sections are also visible at the bottom.



# TESTOVI NORMALNOSTI RASPODELE

U podmeniju **Asses Fit** označiti [Kolmogorov-Smirnov](#) i [Shapiro-Wilk](#)



# TESTOVI NORMALNOSTI RASPODELE

## APA 7 izveštavanje

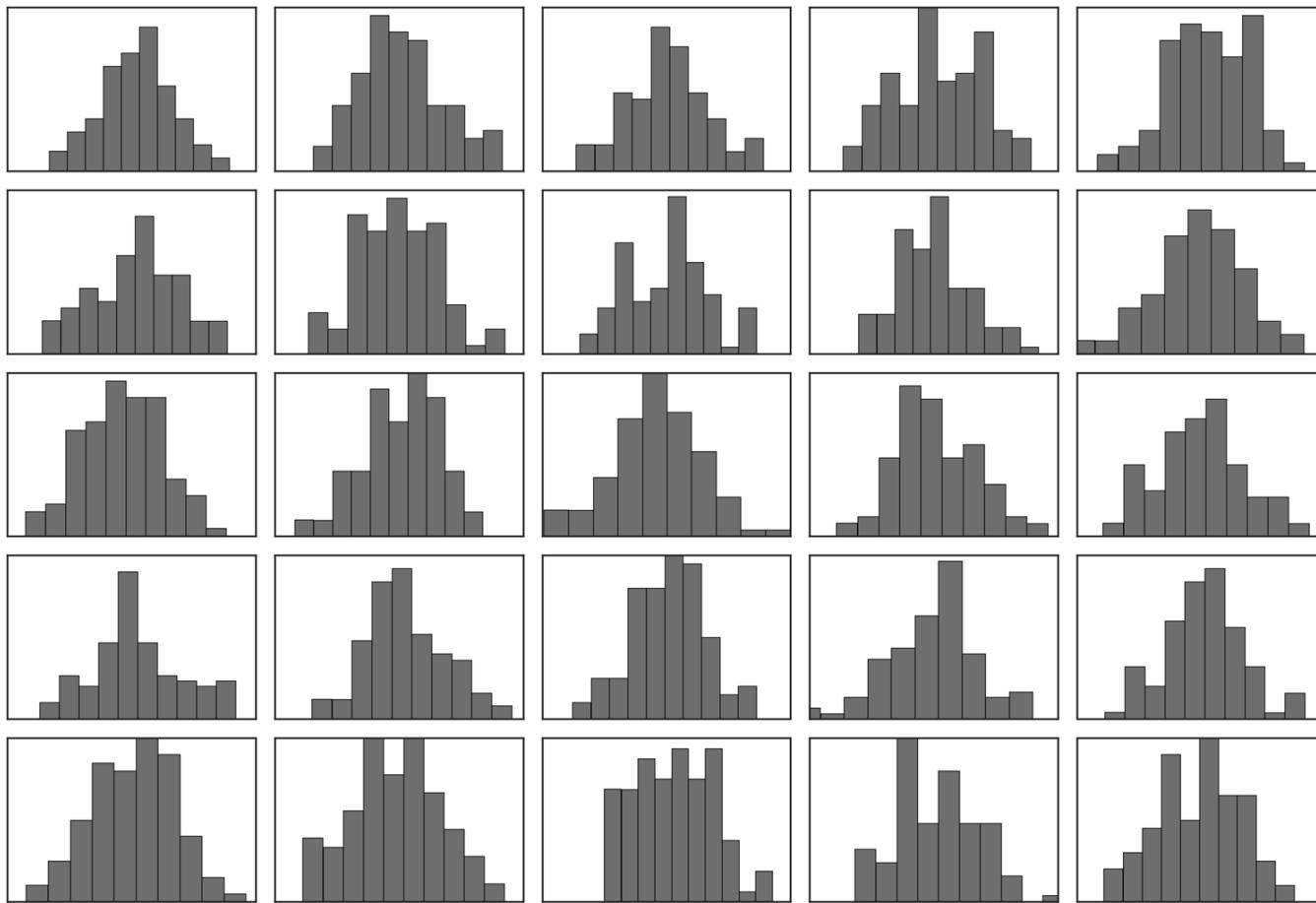


Rezultati Kolmogorov-Smirnov (KS  $Z = .21, p < .001$ ) i Šapiro-Vilk ( $W = .78, p < .001$ ) testa jednoglasno ukazuju na to da empirijska distribucija mera statistički značajno odstupa od modela normalne raspodele.

## Tehnička napomena



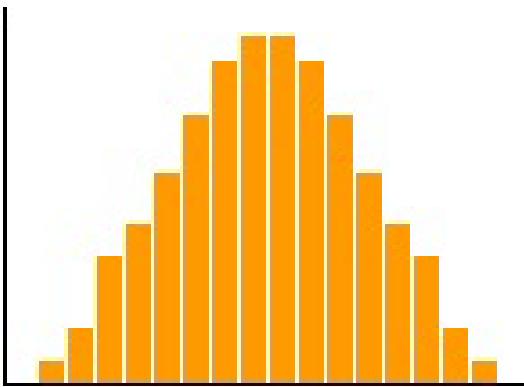
Moguće je da SPSS i JASP daju različite rezultate Kolmogorov-Smirnov testa koji ponekad mogu voditi i sasvim suprotnim zaključcima. Stoga, a i s obzirom na to da ima mnogo veću statističku snagu, preporučuje se upotreba Šapiro-Vilk testa.



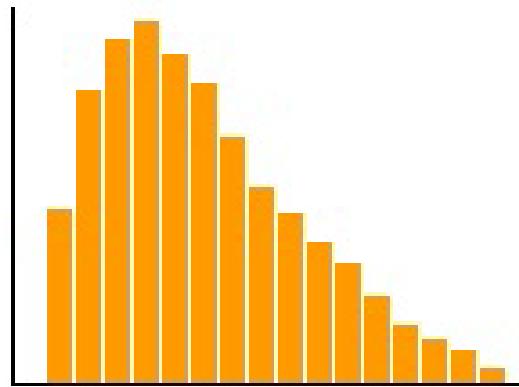
# ODSTUPANJA OD NORMALNE RASPODELE



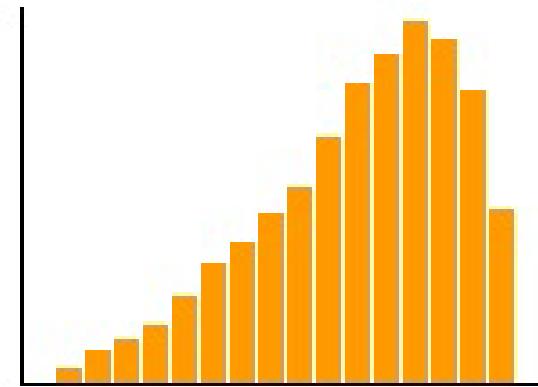
# HORIZONTALNA ASIMETRIJA



Normalna  
raspodela



Pozitivno  
asimetrična



Negativno  
asimetrična

Koncentracija mera je pomerena ka kraju skale.

levom

desnom



# NUMERIČKI POKAZATELJI HORIZONTALNOG ODSTUPANJA

Skjunis (eng. skewness)

$$Sk = \frac{P_{10} + P_{90}}{2} - P_{50}$$

Normalna distribucija: od -0.5 do 0.5  
Negativna zakrivljenost: manje od -0.5  
Pozitivna zakrivljenosti : veće od 0.5

Standardna greška skjunisa

$$SE_{Sk} = \sqrt{\frac{24}{N}}$$

Standandardna greška zavisi od veličine uzorka (veći uzorak, manja greška)

Standardizovani skjunis

$$Z_{Sk} = \frac{Sk}{SE_{Sk}}$$

Vrednost standardizovanog skjunisa govori o značajnosti horizontalnog odstupanja

## Testiranje statističke značajnosti horizontalnog odstupanja

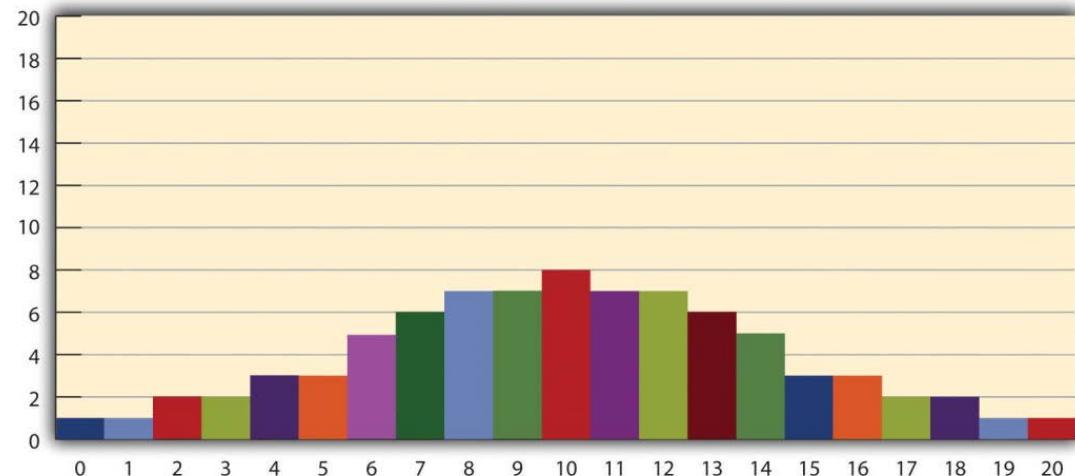
Kada je standardizovani skjunis ( $Z_{Sk}$ )

- |                                  |   |
|----------------------------------|---|
| manji od 1.96, onda je $p > .05$ | tj. odstupanje nije statistički značajno      |
| veći od 1.96, onda je $p < .05$  | tj. odstupanje je značajno na nivou $p < .05$ |
| veći od 2.58, onda je $p < .01$  | tj. odstupanje je značajno na nivou $p < .01$ |

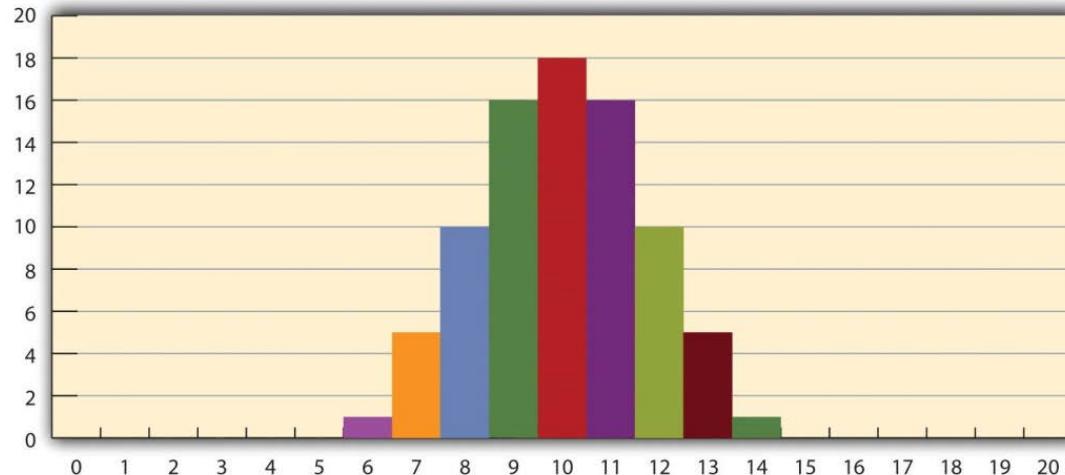


# VERTIKALNA ODSTUPANJA (DEVIJACIJE RASPONA)

Platikurtična  
(spljoštena)



Leptokurtična  
(izdužena)





# NUMERIČKI POKAZATELJI VERTIKALNOG ODSTUPANJA

Kurtozis (eng. skewness)

$$K_u = \frac{P_{75} - P_{25}}{P_{90} - P_{10}}$$

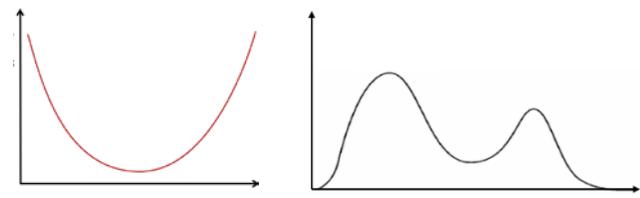
Standardna greška kurtozisa

$$SE_{Ku} = \sqrt{\frac{6}{N}}$$

Standardizovani kurtozis

$$Z_{Ku} = \frac{Ku}{SE_{Ku}}$$

Normalna distribucija: oko nule  
Izduženost (leptokurtičnost): iznad nule  
Spljoštenost (platikurtičnost): ispod nule  
U kriva: ispod -1.2  
Bimodalnost: ekstremno niske vrednosti



Vrednost standardizovanog kurtozisa  
govori o značajnosti vertikalnog  
odstupanja

## Testiranje statističke značajnosti vertikalnog odstupanja

Kada je standardizovani kurtozis ( $Z_{Ku}$ )

manji od 1.96, onda je  $p > .05$  tj. odstupanje nije statistički značajno

veći od 1.96, onda je  $p < .05$  tj. odstupanje je značajno na nivou  $p < .05$

veći od 2.58, onda je  $p < .01$  tj. odstupanje je značajno na nivou  $p < .01$



Analyze → Descriptive Statistics → Descriptives

The screenshot shows the SPSS software interface. The top menu bar includes File, Edit, View, Data, Transform, Analyze, Direct Marketing, Graphs, Utilities, Add-ons, Window, and Help. The Analyze menu is open, with Descriptive Statistics highlighted. A sub-menu under Descriptive Statistics shows several options: Frequencies..., Descriptives... (which is selected and highlighted in yellow), Explore..., Crosstabs..., Ratio..., P-P Plots..., and Q-Q Plots.... To the right of the menu, there is a data view window titled "Visible: 10 of 10 Variables". The data table has columns labeled i5, i6, vreme, ocena, var, var, var, var, var, and var. The rows contain numerical data corresponding to the 10 variables.



# SKJUNIS I KURTOZIS

Prebaciti željenu/e varijable u polje **Variable(s)**.

The screenshot shows the SPSS interface with the 'Data View' tab selected. A 'Descriptives' dialog box is open, displaying the 'Variable(s)' list with 'vreme' and 'ocena' selected. The background data view shows 23 rows of data with columns labeled 'pol', 'smer', 'i1' through 'i6', 'vreme', 'ocena', and several 'var' columns.

	pol	smer	i1	i2	i3	i4	i5	i6	vreme	ocena	var	var	var	var	var	var
1	1	1	4	4	5	1	1	5	.	8						
2	2	5	5	5	5	.	3	3	20	10						
3	2	5	5	5	4	3	3	5	20	10						
4	2	5	2	4	4	2	4	4	20	8						
5	2	5														
6	2	5														
7	2	5														
8	2	.														
9	2	5														
10	2	5														
11	2	2														
12	2	2														
13	2	2														
14	2	2														
15	1	2														
16	2	2														
17	2	2														
18	1	2														
19	2	3														
20	2	3														
21	2	3														
22	2	3														
23	2	2	5	5	5	1	1	5	60	8						



# SKJUNIS I KURTOZIS

U podmeniju Options odabrati Kurtosis i Skewness.

The screenshot shows the SPSS interface with the 'Data View' selected. A 'Descriptives: Options' dialog box is open, centered over the data grid. The dialog box has several sections:

- Mean** and **Sum** tabs (Mean is selected).
- Dispersion** section with options: Std. deviation, Variance, Range, Minimum, Maximum, and S.E. mean.
- Distribution** section with checked boxes for Kurtosis and Skewness.
- Display Order** section with radio buttons for Variable list (selected), Alphabetic, Ascending means, and Descending means.
- Buttons at the bottom: OK, Paste, Reset, Cancel, and Help.
- A checkbox at the bottom left: Save standardized values as variables.

The background data view shows a table with columns labeled 'pol', 'smer', 'i1', 'i2', 'i3', 'i4', 'i5', 'i6', 'vreme', 'ocena', and 'var'. The 'Visible: 10 of 10 Variables' message is displayed in the top right corner of the data view.



# SKJUNIS I KURTOZIS

Screenshot of SPSS software showing Descriptive Statistics output.

The output window displays the following table:

	N	Skewness		Kurtosis	
		Statistic	Std. Error	Statistic	Std. Error
vreme	65	2.270	.297	7.304	.586
ocena	66	-.807	.295	-.267	.582
Valid N (listwise)	63				

Annotations:

- A red arrow points from the text "Skjunis i njegova standardna greška" to the Skewness row.
- A red arrow points from the text "Kurtozis i njegova standardna greška" to the Kurtosis row.

Output Tree:

- Output
- Descriptives
  - Title
  - Notes
  - Descriptive Statistics



# SKJUNIS I KURTOZIS

Pokrenuti meni **Descriptives**

Prebaciti željene varijable u prozor **Variables**

U podmeniju Statistics označiti **Skewness** i **Kurtosis**

The screenshot shows the SPSS Descriptives dialog box. In the top menu bar, the 'Descriptives' icon is highlighted. On the left, the 'Descriptive Statistics' sub-dialog is active, showing variables 'vreme' and 'ocena' selected. The 'Statistics' sub-dialog is open, with 'Skewness' and 'Kurtosis' checked under the 'Distribution' section. A callout box on the right highlights the output table, which displays descriptive statistics for 'vreme' and 'ocena', including skewness and kurtosis values.

	vreme	ocena
Valid	65	66
Missing	3	2
Mean	38.91	8.91
Std. Deviation	29.58	1.22
Skewness	2.27	-0.81
Std. Error of Skewness	0.30	0.29
Kurtosis	7.30	-0.27
Std. Error of Kurtosis	0.59	0.58
Minimum	5.00	6.00
Maximum	180.00	10.00

Vrednosti skjunisa i kurtozisa i njihove standardne greške



# ŠTA KAD RASPODELA ODSTUPA OD NORMALNE?

1. Primeniti odgovarajuću transformaciju varijabli pre standardnih analiza

[Transform → Compute variable](#)

Pozitivno asimetrična distribucija

Logaritamska transformacija (LG10)

Kvadratni koren (SQRT)

Negativno asimetrična distribucija

Ekponencijalna transformacija (EXP)

Stepenovanje (\*\*2, \*\*3, ...)

2. Primeniti odgovarajuće neparametrijske testove

T-test za neponovljena merenja

Man-Vitni test

T-test za ponovljena merenja

Vilkosonov test ekvivalentnih parova

Jednofaktorska ANOVA za neponovljena

Kraskal-Volis test

Jednofaktorska ANOVA za ponovljena

Fridmanov test

Pirsonova korelacija

Spirmanova korelacija